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EFFECTIVE PROPERTIES OF MULTICOMPONENT ELASTOPLASTIC COMPOSITE MATERIALS*

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The present paper generalizes the results obtained in /1/ to the case of an arbitrary number of elastic and elastoplastic components of the medium, by considering the elastoplastic behaviour of a multicomponent composite materials (CM).

1. Consider an elastoplastic, microinhomogeneous medium consisting of n different isotropic components joined to each other with perfect adhesion. Let the first m components be elastoplastic, and the remaining $n-m$ components be perfectly elastic. Hooke's law for such a CM has the form

$$\sigma_{ij}^{(s)} = 2\mu_s(\varepsilon_{ij}^{(s)} - e_{ij}^{p(s)}) + \delta_{ij}\lambda_s\varepsilon_{pp}^{(s)}\ (s = 1, 2, \ldots, m) \tag{1.1}$$

$$\sigma_{ij}^{(s)} = 2\mu_s\varepsilon_{ij}^{(s)} + \delta_{ij}\lambda_s\varepsilon_{pp}^{(s)} \quad (s = m+1, m+2, \ldots, n)$$

Here σ_{ij}, ε_{ij}, e_{ij}^p are the components of the stress, total and plastic deformation tensors, μ_s, λ_s are the Lamé parameters of the component materials, and the plastic deformations satisfy the condition of incompressibility $e_{pp}{}^p = 0$. The plastic properties of the elastoplastic components are described in terms of the Mises yield surface (k_s are the yield points)

$$s_{ij}s_{ij} = k_s^2\ (s = 1, 2, \ldots, m), \quad s_{ij} = \sigma_{ij} - {}^1/_3\,\delta_{ij}\sigma_{pp}$$

The structure of CM can be described by a set of random indicator functions of the coordinates $\varkappa_1(\mathbf{r}), \varkappa_2(\mathbf{r}), \ldots, \varkappa_n(\mathbf{r})$. Every one of these functions $\varkappa_s(\mathbf{r})$ is equal to unity on the set of points of the s-th component, and to zero outside this set. Using these functions we can write the local Hooke's law in the form

$$\sigma_{ij}(\mathbf{r}) = 2\mu(\mathbf{r})(\varepsilon_{ij}(\mathbf{r}) - e_{ij}{}^p(\mathbf{r})) + \delta_{ij}\lambda(\mathbf{r}))\,\varepsilon_{pp}(\mathbf{r}) \tag{1.2}$$

where

$$\mu(\mathbf{r}) = \sum_{s=1}^{n}\mu_s\varkappa_s(\mathbf{r}), \quad \lambda(\mathbf{r}) = \sum_{s=1}^{n}\lambda_s\varkappa_s(\mathbf{r})$$

$$\varkappa_s(\mathbf{r})\,e_{ij}^p(\mathbf{r}) \equiv 0 \quad (s = m+1, m+2, \ldots, n)$$

All functions $\varkappa_s(\mathbf{r})$, stress tensors, total and plastic deformation tensors are assumed to be statistically homogeneous and ergodically random fields, and their expectations are replaced by the following quantities /2/ averaged over the component volumes V_s and over the whole volume V of the medium:

$$\langle f\rangle = \frac{1}{V}\int_V f(\mathbf{r})\,d\mathbf{r},\ \langle f\rangle_s = \frac{1}{V_s}\int_{V_s} f(\mathbf{r})\,d\mathbf{r} \quad \left(V = \sum_{s=1}^{n} V_s\right)$$

Supplementing relation (1.2) with the equations of equilibrium $\sigma_{ij,j}(\mathbf{r}) = 0$ and the Cauchy formulas $2\varepsilon_{ij}(\mathbf{r}) = u_{i,j}(\mathbf{r}) + u_{j,i}(\mathbf{r})$ connecting the components of the total deformation tensor with the components of the displacement vector $u_i(\mathbf{r})$, we obtain a closed system of equations describing the deformation of a multicomponent CM whose boundary conditions are that there are no fluctuations in the value of the quantities on the surface S of the volume V

$$f(\mathbf{r})\big|_{\mathbf{r}\in S} = \langle f\rangle$$

Using Green's tensor

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$$G_{ik}(\mathbf{r}) = \frac{1}{8\pi\langle\mu\rangle}\left(\delta_{ik}r_{,pp} - \frac{\langle\lambda\rangle + \langle\mu\rangle}{\langle\lambda\rangle + 2\langle\mu\rangle} r_{,ik}\right); \quad r = |\mathbf{r}|$$

we can reduce the above system of equations to a system of integral equations of equilibrium /1/

$$\varepsilon_{ij}'(\mathbf{r}) = \int_V G_{ik,lj}(\mathbf{r}-\mathbf{r}_1)(2\langle\mu\rangle e_{ki}^p(\mathbf{r}_1) + \chi_{ki}'(\mathbf{r}_1)\,d\mathbf{r}_1 \tag{1.3}$$

$$\chi_{ij}(\mathbf{r}) = -\sum_{s=1}^{n}(2\mu_s(\varepsilon_{ij}(\mathbf{r}) - e_{ij}^p(\mathbf{r})) + \delta_{ij}\lambda_s\varepsilon_{pp}(\mathbf{r})\varkappa'(\mathbf{r})$$

(the primes denote the fluctuations in the quantities over the whole volume V of the CM).

In order to find the effective constants of the CM in question, we must establish the relation connecting the macroscopic stresses and deformations. Let us average (1.2) over the whose volume V of the body, and apply the rule of mechanical mixing of phases

$$\langle\sigma_{ij}\rangle = \sum_{s=1}^{n} c_s(2\mu_s\langle\varepsilon_{ij}\rangle_s + \delta_{ij}\lambda_s\langle\varepsilon_{pp}\rangle_s) - \sum_{s=1}^{m} 2\mu_s c_s\langle e_{ij}^p\rangle_s \tag{1.4}$$

where $c_s = \langle\varkappa_s\rangle = V_sV^{-1}$ are the volume contents of the components. Eq.(1.4) shows that in order to establish the rheological macroscopic law we must calculate the deformations averaged over the component volumes. The quantities $\langle\varepsilon_{ij}\rangle_s$ can be found from the well-known relations /3/

$$\langle\varepsilon_{ij}\rangle_s = \langle\varepsilon_{ij}\rangle + c_s^{-1}\langle\varkappa_s'\varepsilon_{ij}'\rangle \tag{1.5}$$

Let us calculate the quantities $\langle\varkappa_q'\varepsilon_{ij}'\rangle$ $(q = 1, 2, \ldots, n)$, restricting ourselves to the singular approximation /1, 2/. We multiply Eq.(1.4) by $\varkappa_q'(\mathbf{r})$ and average it over the whole volume V

$$\langle\varkappa_q'\varepsilon_{ij}'\rangle = \int_V G_{ik,lj}(\mathbf{r}_1)(2\langle\mu\rangle\langle\varkappa_q'(\mathbf{r})e_{kl}^{p'}(\mathbf{r}+\mathbf{r}_1)\rangle + \langle\varkappa_q'(\mathbf{r})\chi_{kl}'(\mathbf{r}+\mathbf{r}_1)\rangle)\,d\mathbf{r}_1$$

Neglecting, in accordance with the hypothesis of singular approximation, the formal parts of the second derivatives of Green's tensor, we obtain /1/

$$\langle\varkappa_q'\varepsilon_{ij}'\rangle = (\alpha I_{ijkl} - \beta\delta_{ij}\delta_{kl})\Big(\sum_{s=1}^{m} 2\mu_s\langle\varkappa_q'\varkappa_s e_{kl}^p\rangle - \sum_{s=1}^{n}(2\mu_s\langle\varkappa_q'\varkappa_s'\varepsilon_{kl}\rangle +$$
$$\delta_{kl}\lambda_s\langle\varkappa_q'\varkappa_s'\varepsilon_{pp}\rangle)\Big)(2\langle\mu\rangle)^{-1}$$
$$\alpha = \frac{2}{15}\,\frac{4-5\langle\nu\rangle}{1-\langle\nu\rangle}, \quad \beta = \frac{1}{15(1-\langle\nu\rangle)}, \quad \nu_s = \frac{\lambda_s}{2(\mu_s+\lambda_s)}$$

Separating from this relation the deviator and volume parts, we obtain

$$\langle\varkappa_q'e_{ij}'\rangle = \alpha\Big(\sum_{s=1}^{m} m_s\langle\varkappa_q'\varkappa_s e_{ij}^p\rangle - \sum_{s=1}^{n} m_s\langle\varkappa_q'\varkappa_s'e_{ij}\rangle\Big) \tag{1.6}$$

$$\langle\varkappa_q'\varepsilon_{pp}'\rangle = 3\,\frac{\alpha-3\beta}{2\langle\mu\rangle}\sum_{s=1}^{n} K_s\langle\varkappa_q'\varkappa_s'\varepsilon_{pp}\rangle$$

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{pp}, \quad K_s = 2\mu_s + 3\lambda_s, \quad m_s = \frac{\mu_s}{\langle\mu\rangle}$$

Eliminating the quantities $\langle\varkappa_q'\varepsilon_{ij}'\rangle$ from the equations (1.5), (1.6) and taking into account the relations

$$\langle\varkappa_q'\varkappa_s'f\rangle = \begin{cases} c_qc_s(\langle f\rangle - \langle f\rangle_q - \langle f\rangle_s), & q \neq s \\ c_s(c_s\langle f\rangle + (1-2c_s)\langle f\rangle_s, & q = s\end{cases}$$

$$\langle\varkappa_q'\varkappa_s f\rangle = \begin{cases} -c_qc_s\langle f\rangle_s, & q \neq s \\ c_s(1-c_s)\langle f\rangle_s, & q = s\end{cases}$$

we obtain

$$\langle e_{ij}\rangle_s = \left((1-\alpha)\langle e_{ij}\rangle + \alpha m_s\langle e_{ij}^p\rangle_s + \alpha\frac{\langle s_{ij}\rangle}{2\langle\mu\rangle}\right)(1+\alpha(m_s-1))^{-1} \tag{1.7}$$

$$\langle\varepsilon_{pp}\rangle_s = \left((1-\gamma)\langle\varepsilon_{pp}\rangle + \gamma\frac{\langle\sigma_{pp}\rangle}{3\langle K\rangle}\right)(1+\gamma(q_s-1))^{-1}$$

$$\gamma = \frac{1}{3}\,\frac{1+\langle\nu\rangle}{1-\langle\nu\rangle}, \quad q_s = \frac{K_s}{\langle K\rangle}$$

Substituting relations (1.7) into (1.4) and separating the resulting expression into the deviator and volume parts, we obtain the macroscopic Hooke's law for the CM under consideration

$$\langle s_{ij}\rangle = 2\mu^* (\langle e_{ij}\rangle - e_{ij}^*), \quad \langle \sigma_{pp}\rangle = 2K^* \langle \varepsilon_{pp}\rangle \tag{1.8}$$

$$\mu^* = \langle\mu\rangle \frac{1-(1-\alpha)\langle a\rangle}{\alpha\langle a\rangle}, \quad K^* = \langle K\rangle \frac{1-(1-\gamma)\langle b\rangle}{\gamma\langle b\rangle}, \quad \langle a\rangle = \sum_{s=1}^{n} c_s a_s$$

$$\langle b\rangle = \sum_{s=1}^{n} c_s b_s, \quad a_s = \frac{1}{1+\alpha(m_s-1)}, \quad b_s = \frac{1}{1+\gamma(q_s-1)}$$

Here μ^*, K^* are the effective shear and bulk moduli, and e_{ij}^* are the components of the residual deformation tensor of the CM, which are measured after the external loads have been removed from the surface of the body. They are connected with the plastic deformations averaged over the component volumes by the relations

$$e_{ij}^* = \frac{\langle\mu\rangle}{\mu^*\langle a\rangle} \sum_{s=1}^{m} c_s m_s a_s \langle e_{ij}^p\rangle_s \tag{1.9}$$

The expressions for the effective moduli of elasticity appearing in Hooke's law are the same as the well-known formulas of the singular approximation in the theory of elasticity of microinhomogeneous multicomponent media /2/.

2. Let us consider the macroscopic behaviour of a multicomponent CM beyond the elastic limit. To be specific, we shall assume without loss of generality, that the material of the first component becomes plastic first, then that of the second component, etc., up to and including the m-th component. Let the plastic flow develop in the first stage within the volume V_1 only. Averaging the local yield surface over the volume V_1 of the first component and using the condition that the square of the mean is always less than the mean of the square, we obtain an upper estimate for the yield surface within the volume V_1 /1/

$$\langle s_{ij}\rangle_1 \langle s_{ij}\rangle_1 \leqslant k_1^2 \tag{2.1}$$

Using Hooke's law for the first component to eliminate the components of the stress tensor deviator s_{ij} from (2.1), we obtain

$$4\mu_1^2 \langle e_{ij} - e_{ij}^p\rangle_1 \langle e_{ij} - e_{ij}^p\rangle_1 \leqslant k_1^2 \tag{2.2}$$

Substituting the formulas (1.7)-(1.9) into inequality (2.2), we obtain an upper estimate for the macroscopic load surface

$$(\langle s_{ij}\rangle - N_1 e_{ij}^*)(\langle s_{ij}\rangle - N_1 e_{ij}^*) \leqslant k_1^{*2}$$

and the associated rule of flow corresponding to this surface

$$\langle s_{ij}\rangle = k_1^* \frac{\dot{e}_{ij}^*}{\sqrt{\dot{e}_{kl}^* \dot{e}_{kl}^*}} + N_1 e_{ij}^*, \quad \dot{e}_{ij}^* = \frac{de_{ij}^*}{dt}, \quad k_1^* = \xi_1 \frac{k_1}{m_1} \tag{2.3}$$

$$N_1 = \frac{2\mu_1\xi_1}{c_1 m_1}\left[\frac{\xi_1}{m_1} - c_1 a_1 (1+\alpha(\xi_1 - 1))\right], \quad \xi_s = \frac{\mu^*\langle a\rangle}{\langle\mu\rangle a_s}$$

(k_1^* is the effective yield point and N_1 is the coefficient of linear kinematic hardening). In this case Eq.(1.9) takes the form

$$\langle e_{ij}^p\rangle = h_1 e_{ij}^*, \quad h_1 = \xi_1/m_1$$

Relation (2.3) represents the law governing the flow of a plastic body with linear kinematic hardening.

The macroscopic behaviour of the CM will correspond to Eq.(2.3) until the second component reaches its critical state in which plastic deformations have not yet occurred, but the stresses already obey the Mises yield conditions. The upper estimates for this state will be given by the conditions

$$\langle s_{ij}\rangle_2 \langle s_{ij}\rangle_2 \leqslant k_2^2, \quad \langle e_{ij}^p\rangle_2 = 0 \tag{2.4}$$

Substituting into the first condition of (2.4) Hooke's law for the second component and the formulas (1.7)-(1.9), we obtain

$$(\langle s_{ij}\rangle + 2\mu^*(1-\alpha)\langle a\rangle e_{ij}^*)(\langle s_{ij}\rangle + 2\mu^*(1-\alpha)\langle a\rangle e_{ij}^*) \leqslant k_2^2\xi_2^2/m_2^2 \tag{2.5}$$

The intersection of the load surface with the surface (2.5) in six-dimensional macrostress space, yields the values of the residual deformation $\omega_{ij}^{(1)}$ and the macroscopic stresses $\tau_{ij}^{(1)}$ determining the limits of the correspondence between the associated law of flow (2.3) and the macroscopic behaviour of the CM

$$\omega_{ij}^{(1)} = \eta_1 \frac{\dot{e}_{ij}^*}{\sqrt{\dot{e}_{kl}^* \dot{e}_{kl}^*}}, \quad \eta_1 = \frac{k_2\xi_2 m_2^{-1} - k_1^*}{N_1 + 2\mu^*(1-\alpha)\langle a\rangle} \tag{2.6}$$

$$\tau_{ij}^{(1)} = (k_1^* + \eta_1 N_1)\frac{\dot{e}_{ij}^*}{\sqrt{\dot{e}_{kl}^* \dot{e}_{kl}^*}}$$

Let us consider the next deformation stage, when the plastic flow develops in the first, as well as the second phase, and the remaining components remain perfectly elastic. Let us average simultaneously the local yield surfaces over the volumes of the first and second components V_1, V_2, and use Hooke's law

$$4\mu_q^2 \langle e_{ij} - e_{ij}^p \rangle_q \langle e_{ij} - e_{ij}^p \rangle_q \leqslant k_q^2 \quad (q = 1,2) \tag{2.7}$$

Substituting the formulas (1.7)-(1.9) into the inequalities (2.7) and using the rule of mixtures, we obtain the upper estimate for the macroscopic load surface for which the corresponding associated law of flow has the form

$$\langle s_{ij} \rangle = k_2^* \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}} + 2\mu^* \frac{\langle a \rangle}{\{a\}_2} \left[\langle e_{ij}^p \rangle - \left(\alpha \langle a \rangle \frac{\mu^*}{\langle \mu \rangle} + (1-\alpha) \{a\}_2 \right) e_{ij}^* \right] \tag{2.8}$$

$$k_2^* = \mu^* \langle a \rangle \left\{ \frac{k}{m} \right\}_2 (\langle \mu \rangle \{a\}_2)^{-1}, \quad \{f\}_q = \sum_{s=1}^{q} c_s f_s$$

(the braces denote averaging of the quantities over the set of component volumes). Substituting the inequality

$$\langle e_{ij}^p \rangle \leqslant h_2 e_{ij}^*, \quad h_2 = \max_{i,j} \left\{ \frac{\langle e_{ij}^p \rangle}{e_{ij}^*} \right\} \tag{2.9}$$

(no summation over i, j) into Eq.(2.8), strengthens the upper estimate of the macroscopic behaviour of the CM. Eliminating the quantities $\langle e_{ij}^p \rangle$ and h_2 from the formulas (2.6), (2.8) and (2.9), we obtain

$$\langle s_{ij} \rangle = k_2^* \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}} + N_2 e_{ij}^* \tag{2.10}$$

$$N_2 = N_1 - \frac{k_2^* - k_1^*}{\eta_1}$$

The values of the residual deformations $\omega_{ij}^{(2)}$ and macrostresses $\tau_{ij}^{(2)}$ determining the boundaries of the correspondence between the effective behaviour of the CM and Eq.(2.10), are found from the condition analogous to the inequality (2.4)

$$\langle s_{ij} \rangle_3 \langle s_{ij} \rangle_3 \leqslant k_3^2, \quad \langle e_{ij}^p \rangle_3 = 0$$

and are equal to

$$\omega_{ij}^{(2)} = \eta_2 \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}}, \quad \tau_{ij}^{(2)} = (k_2^* + \eta_2 N_2) \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}} \tag{2.11}$$

$$\eta_2 = \frac{k_3 \xi_3 m_3^{-1} - k_2^*}{N_2 + 2\mu^* (1-\alpha) \langle a \rangle}$$

Repeating the previous arguments, we can obtain the law governing the deformation of the CM for the case when the third, fourth, etc. component becomes plastic. In the general case, when the plastic flow develops in the first q $(q < m)$ components, the upper estimate for the effective law of flow in the CM has the form

$$\langle s_{ij} \rangle = k_q^* \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}} + N_q e_{ij}^* \tag{2.12}$$

$$k_q^* = \left\{ \frac{k}{m} \right\}_q \mu^* \langle a \rangle (\langle \mu \rangle \{a\}_q)^{-1}, \quad N_q = N_{q-1} - \frac{k_q^* - k_{q-1}^*}{\eta_{q-1}}$$

The boundaries of the correspondence between Eq.(2.12) and the macroscopic behaviour of the CM are given by the expressions

$$\omega_{ij}^{(q)} = \eta_q \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}}, \quad \tau_{ij}^{(q)} = (k_q^* + \eta_q N_q) \frac{e_{ij}^{\cdot *}}{\sqrt{e_{kl}^{\cdot *} e_{kl}^{\cdot *}}} \tag{2.13}$$

$$\eta_q = \frac{k_{q+1} \xi_{q+1} m_{q+1}^{-1} - k_q^*}{N_q + 2\mu^* (1-\alpha) \langle a \rangle}$$

Thus the behaviour of the CM beyond the elastic limit is estimated from above by the law of flow of a plastic body with the kinematic, piecewise linear hardening. When $n = 2, m = 1$, the general formulas (2.12) are identical with the analogous results for the two-component media obtained in /1/.

3. In order to apply the above scheme for determining the properties of the CM in practice, we must know in what order the components of the CM pass to the plastic state. Let us establish the dependence of this order on the mechanical constants of the component materials

and their volume contents. We will consider two arbitrary volumes V_p, V_q which are in the elastic state. We introduce the quantity

$$\Delta_{pq} = \frac{\langle s_{ij}s_{ij}\rangle_q}{k_q^2} - \frac{\langle s_{ij}s_{ij}\rangle_p}{k_p^2} \tag{3.1}$$

It is clear that if $\Delta_{pq} > 0$, then the plastic flow will first occur in V_q, while when $\Delta_{pq} < 0$, plastic deformations will first appear in the volume V_p. The condition $\Delta_{pq} = 0$ means that both components become plastic simultaneously (condition of equiplasticity) /4/.

Let us express Δ_{pq} in terms of the macroscopic stresses and residual deformations. Since there are no plastic deformations within the volumes V_p, V_q, it follows that

$$\langle s_{ij}s_{ij}\rangle_s = 4\mu_s^2 \langle e_{ij}e_{ij}\rangle_s \quad (s = p, q) \tag{3.2}$$

We determine the quantity $\langle e_{ij}e_{ij}\rangle_s$ by multiplying (1.3) by $\varkappa_s'(\mathbf{r})\, e_{ij}(\mathbf{r})$ and averaging the results over the total volume V of the composite material. Applying to the integral obtained the hypothesis of singular approximation and carrying out calculations analogous to those used in deriving Eq.(1.6), we obtain

$$\langle e_{ij}e_{ij}\rangle_s = \frac{\zeta_{ij}\zeta_{ij}}{[1 + \alpha(m_s - 1)]^2} \tag{3.3}$$

$$\zeta_{ij} = \left(\frac{\alpha}{2\langle\mu\rangle} + \frac{1-\alpha}{2\mu^*}\right)\langle s_{ij}\rangle + (1-\alpha)\, e_{ij}^*$$

Substituting (3.2), (3.3) into (3.1), we find that if

$$\left|\frac{\mu_q}{k_q(1 + \alpha(m_q - 1))}\right| \leqslant \left|\frac{\mu_p}{k_p(1 + \alpha(m_p - 1))}\right|$$

then the plastic flow will first begin within the volume V_p. The equality corresponds to the condition that plastic flow occurs in the materials of both components simultaneously.

Expanding the quantities

$$\left|\frac{\mu_s}{k_s(1 + \alpha(m_s - 1))}\right| \quad (s = 1, 2, \ldots, m)$$

in the order in which they decrease, we obtain the order in which the component materials become plastic.

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